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ABSTRACT

Fifty-six Dutch school children (aged 6-0 to 14-4 years) participated in a study designed to investigate their ability to reason with logical implication. They answered a total of 32 reasoning problems. Before or after each question they were asked a class inclusion question. A high degree of correspondence was found between class inclusion and reasoning with logical implication premises. The child's understanding of the subordinate and superordinate characteristics of the premises appeared to be a decisive factor in judging the truth of logical arguments, although many correct judgments were based on incorrect understanding of the premises. It was concluded that when children fail to solve a problem, they do not necessarily do so because they lack the particular logical operations, but because of a misunderstanding of the premises. Incomplete analysis of the problem appeared to be a second source of error. Further, understanding of logical premises was found to be improving with age. (Author)

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CHILDREN'S UNDERSTANDING OF CLASS INCLUSION AND THEIR ABILITY TO REASON WITH IMPLICATION

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Children's Understanding of Class Inclusion and their Ability to Reason with Implication

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ABSTRACT

Fifty-six Dutch school children (aged 6-0 to 14-4 years) participated in a study designed to investigate their ability to reason with logical implication. They answered a total of 32 reasoning problems. Before or after each question they were asked a class inclusion question. A high degree of correspondence was found between class inclusion and reasoning with logical implication premises. The child's understanding of the subordinate and superordinate characteristics of the premises appeared to be a decisive factor in judging the truth of logical arguments, although many correct judgments were based on incorrect understanding of the premises. It was concluded that when children fail to solve a problem, they do not necessarily do so because they lack the particular logical operations, but because of a misunderstanding of the premises. Incomplete analysis of the problem appeared to be a second source of error. Further, understanding of logical premises was found to be improving with age.

Introduction

The preoperational child is described by Piaget (1952, 1953; Inhelder, & Piaget, 1958) as finding it difficult to think about part-whole relationships. This has been established in a number of empirical studies directed at children's class inclusion reasoning (see Winer, 1980, for a review). Although the child's performance on the class inclusion task has been conceived in the past as a simple indicator of the onset of operational thought (e.g., Inhelder, & Piaget, 1964), most researchers nowadays contend that class inclusion requires a variety of complex skills, including perceptual, semantic, linguistic, and quantitative operations (e.g., Trabasso, Isen, Dolecki, McLanahan, Riley, & Tucker, 1978; Winer, 1980). An enormous quantity of information is available now on class inclusion; however, it still is not very well understood what role the understanding of inclusion relations plays in logical problem solving behavior in general. The present study focuses on the class inclusion aspects of implication reasoning.

Of the sixteen binary operations that, according to Piaget (1953) form the basis of mature formal operational thought, implication is by far the most thoroughly investigated (e.g., Ennis, 1971; Johnson-Laird, & Tagart, 1969; Kodroff, & Roberge, 1975; Kuhn, 1977; Mason, Bramble, & Mast, 1975; Paris, 1973; Roberge, & Mason, 1978; Taplin, Staudenmayer, & Taddonio, 1974; Wason, & Johnson-Laird, 1972), indicating that it is believed by many to be central in human reasoning. As has been discussed by Ennis (1975) the logic of implication and of class inclusion share identical propositions. That is, $p.q$, $\bar{p}.q$, $\bar{p}.\bar{q}$,

and $N(p.\bar{q})$ [1] In terms of classes, implication corresponds to $P<Q$. Thus, the statement that "X is a dog" implies that "X is an animal". This is true because all dogs are animals. The class of dogs is completely contained in the class if animals. Therefore, "if p, then q" can be regarded as logically equivalent to "all P are Q". The relationship between class inclusion and the logic of implication suggests that children must be able to recognize superordinate and subordinate relationships before mature implication reasoning can be demonstrated. Further, one would expect incorrect understanding of the inclusion relation to adversely affect implication reasoning.

In the study of implication reasoning wide use has been made of formal syllogisms (e.g., Roberge, 1970; Roberge, & Mason, 1978; Shapiro, & O'Brien, 1970; Wason, & Johnson-Laird, 1972). Usually, the subject is presented with premises and conclusion, and is asked to indicate whether or not the particular argument form is valid. It is, however, questionable whether the responses obtained in these tasks provide useful information on the logical reasoning that occurs: one may falsely interpret the absence of correct responses as indicating the absence of the process, and, on the other hand, their presence as indicating the presence of the process (see Smedslund, 1969). Some authors suggest that subjects in judging the truth of logical arguments often misinterpret the premises of the problem (Ceraso, & Provitera, 1971; Henle, 1962; Revlin,

[1] In each of the above, p is subordinate to q, a line over a statement or element indicates negation, a dot indicates conjunction, and N stands for negation. Therefore, $\bar{p}.\bar{q}$ should be read as "not p and not q".

Ammerman, Petersen, & Leirer, 1978). The view taken in this paper is that in spite of misunderstanding of the premises of a problem, the subject may still behave systematically and according to logical rules. Hence, it seems useful to attempt to uncover his or her understanding of the premises of the task, rather than merely assessing the correctness of the response from the point of view of formal logic.

The relation between the classes contained in implication statements may be understood in basically five different ways. For example, the premise "If all dogs are animals" may be understood by the child to mean any of the following:

1. dogs and animals are part of the same concept (equivalence)
2. dogs and animals both exist as separate concepts (exclusion)
3. some dogs are animals and some animals are dogs (intersect)
4. animals are part of the group of dogs (inverse inclusion)
5. dogs are part of the group of animals (inclusion)

Clearly, only the fifth category reflects correct understanding of the relation that exists between the two classes. It is hypothesized here that judgments of the truth of conditional arguments can be predicted from a knowledge of the subject's understanding of the premises. The specific predictions for four argument forms are shown in Table 1.

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Insert Table 1 here

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In the present study the following hypotheses were inves-

tigated:

1. Understanding of the class relations as expressed by conditional arguments will be related to class inclusion performance.

2. Children's judgments of the truth of conditional arguments will be based on their understanding of the premises.

3. There will be developmental differences across ages in children's understanding of logical premises and syllogisms containing implication.

Methods

Subjects.

The 56 subjects who participated in the study were pupils in kindergarten and second, fourth, sixth, and eighth grade in Dutch schools in a city in eastern Holland. Average ages were: 6-0 (kgn), 8-1 (second), 10-2 (fourth), 12-2 (sixth), and 14-4 (eighth). There were 29 boys and 27 girls in the sample.

Materials.

As part of a larger study in reasoning, children were administered reasoning problems individually in an interview format. Each of four syllogistic arguments (Modus Ponens (MP), Modus Tollens (MT), Denial of the Antecedent (DA), and Affirmation of the Consequent (AC)) was presented eight times in the interview. The conditional arguments were not only read to the child, but were demonstrated using pictures, or toy animals or blocks. Children were asked whether or not they thought the

arguments were valid using the same three-choice response paradigm (YES, NO, or MAYBE) that has been used in other reasoning research (Ennis, & Paulus, 1965; Roberge, 1970). The interview, its construction, reliability, and other relevant information has been described elsewhere (Mason, Haars, Dingemans, & Bierman-van Duuren, 1979).

Procedures.

Three practice items were given which did not use the same arguments as the remainder of the study to insure that the subject understood how to respond. After each subject responded, he or she was asked to explain the answer. For each major premise of the argument the child was asked to respond to, there was a class inclusion question. For example, if the major premise was: "if all dogs are animals", the class inclusion question would be: "are there more dogs or animals ?" Half of the items were presented with the class inclusion question preceding the argument, and half with the inclusion question following. Further, children were asked to explain both the minor premise combined with the conclusion, and of the major premise of each of the arguments.

For scoring verbal responses a scoring system was designed which was directed at the child's understanding of the subordinate and superordinate characteristics of the terms in the premises.

All of the interviews were conducted in the children's native language by Dutch graduate students in psychology from the university of Nijmegen. Interviews were recorded on cassette tapes and scored later.

Results

The frequencies of responses to the class inclusion questions and children's explanations of the conditional arguments are shown in Table 2.

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Insert Table 2 here

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Chi-square analysis yielded highly significant results ($p<.001$ for $\chi^2=350.88$, $df=8$). As can be seen in the Table the class inclusion questions were very frequently answered correctly, and children who did so also tended to give correct explanations of the conditional arguments. The next frequent were the subordinate answers ("more P"). This was true in all categories, except for equivalence , where "P=Q" was the second most frequent answer to the class inclusion question.

Table 3 shows the categories of explanation of the major premise and the frequencies of validity judgments for each of the four argument forms.

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Insert Table 3 here

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Significant results were obtained on all four argument forms ($p<.001$ for $\chi^2=70.22$ with $df=8$ for MP; $p<.001$ for $\chi^2=66.59$ with $df=8$ for MT; $p<.001$ for $\chi^2=45.67$ with $df=8$ for DA; and $p<.001$ for $\chi^2=123.36$ with $df=8$ for AC). Further analysis using Ryan's (1960) procedure (with $\alpha=.05$) revealed the following: Modus

Ponens was most often judged to be invalid in a class exclusion interpretation, while it was regarded valid in relation to identity and inclusion interpretations. Modus Tollens was thought to be invalid by subjects who interpreted the classes in the major premise as either having an identity or an inclusion relation. Denial of the Antecedent was judged to be invalid in an identity interpretation. And, finally, Affirmation of the Consequent was considered to be invalid in an exclusion interpretation, and valid in an identity interpretation, while the answer "maybe" was most often given in the inclusion category.

The proportions of responses in each of the categories of understanding and for each grade are shown in Table 4.

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Insert Table 4 here

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As can be seen, there appears to be a decrease across ages in those categories that reflect incorrect understanding. Further, consistent and clear-cut age differences can be observed in the inclusion category. In addition, there appears to be a marked decline in the "no response" category. In all categories, differences are especially large between ages six and eight.

Discussion

The results indicate that responses to class inclusion questions correspond to specific categories of understanding. This finding suggests that the child's knowledge of the objects

mentioned is a major factor in solving class inclusion problems: younger children may use the terms to mean something different from the more mature speaker. The data show that the understanding of the class inclusion aspects of implication improves at successive age levels.

Further, the results provide additional evidence in favor of earlier observations that class inclusion plays an important part in conditional reasoning (e.g., Kuhn, 1977): understanding of inclusion was highly related to provision of correct judgments. In addition, judgments appeared to be predictable, even in those cases where the inclusion relation was misunderstood. It follows that the reasoning process underlying both correct and incorrect responses was highly systematic and consistent. Thus, to say that a child who fails to give correct solutions does not show the proper logic seems to be unwarranted.

However, in some cases the predicted response was not often given. The "maybe" answer was generally avoided in all four argument forms. This answer is expected to be given when two correct but contrasting solutions are possible (i.e., "YES" and "NO"). For instance, from the premises "if all dogs are animals" and "this is an animal" one may conclude "it might or might not be a dog". Thus, the absence of the answer indicates that the child probably does not recognize both possibilities. In other words, although the child understands the premises in such a way as to allow for both possibilities, only one of them is actually chosen as the correct solution. It seems as if the problem is incompletely analysed: the analysis is stopped prematurely whenever a possible solution is encountered. This

is not very different from observations made by others (see Ceraso, & Provitera, 1971; Neimark, 1970; Neimark, & Chapman, 1975). It is clear that the marked and consistent differences in difficulty often described in the empirical literature between valid and invalid arguments can --at least in part-- be explained by this incomplete analysis effect.

In conclusion, the study presented here provides evidence that young children can and do more reasoning than is usually recognized. Their reasoning appears to be more systematic and consistent than is often thought. However, the task received by the child and the one intended by the examiner are not necessarily isomorphic. The researcher should realize that children sometimes give the right answers for the wrong reasons, while their incorrect answers are often a logical consequence of their way of understanding. The young child is often depicted as lacking in reasoning skills, and as an imperfect thinker who uses a defective logic. There is no doubt that preoperational or concrete operational children cannot do many things, but it should also be recognized that there are things they can do, sometimes very well.

References

Ceraso, J. & Provitera, A. Sources of error in syllogistic reasoning. Cognitive Psychology, 1971, 2, 400-410.

Ennis, R.H. Children's ability to handle Piaget's propositional logic: a conceptual critique. Review of Educational Research, 1975, 45, 1-41.

Ennis, R.H. Conditional logic and primary school children. Interchange, 1971, 2, 126-132.

Ennis, R.H. & Paulus, D. Critical thinking readiness in grades 1-12. Phase 1: Deductive reasoning in adolescence. Ithaka, N.Y.: Cornell University, 1965. (Eric Reproduction Service, No ED 003 818).

Henle, M. On the relation of logic and thinking. Psychological Review, 1962, 69, 366-378.

Inhelder, B. & Piaget, J. The early growth of logic in the child: classification and seriation. New York: Humanities Press, 1964.

Inhelder, B. & Piaget, J. The growth of logical thinking from childhood to adolescence. New York: Basic Books, 1958.

Johnson-Laird, P.N. & Tagart, J. How implication is understood. American Journal of Psychology, 1969, 82, 367-373.

Kodroff, J.K. & Roberge, J.J. Developmental analysis of the conditional reasoning abilities of primary-grade children. Developmental Psychology, 1975, 11, 21-28.

Kuhn, D. Conditional reasoning in children. Developmental Psychology, 1977, 13, 342-353.

Mason, E.J., Bramble, W.J., & Mast, T.A. Familiarity with content and conditional reasoning. Journal of Educational Psychology, 1975, 67, 238-242.

Mason, E.J., Haars, V.J.E., Dingemans, P.J.M., & Bierman-van Duuren, P. Two structured interviews for studying development of logical reasoning in children and adolescents. Internal Report, Katholieke Universiteit Nijmegen, the Netherlands, 1979.

Neimark, E.D. Development of comprehension of logical connectives: understanding of "or". Psychonomic Science, 1970, 21, 217-219.

Neimark, E.D., & Chapman, R.H. Development of the comprehension of logical quantifiers. In R.H. Falmagne, Reasoning: representation and process in children and adults. Hillside, N.J.: Erlbaum, 1975.

Paris, S.G. Comprehension of language connectives and logical relationships. Journal of Experimental Child Psychology, 1973, 16, 278-291.

Piaget, J. Logic and Psychology. London: Manchester University Press, 1953.

Piaget, J. The child's conception of number. New York: Humanities Press, 1952.

Revlin, R., Ammerman, K., Peterson, K., & Leirer, V. Category relations and syllogistic reasoning. Journal of Educational Psychology, 1978, 70, 613-625.

Roberge, J.J. A study of children's abilities to reason with basic principles of deductive reasoning. American Educational Research Journal, 1970, 7, 583-596.

Roberge, J.J., & Mason, E.J. Effect of negation on adolescent's class and conditional reasoning abilities. Journal of General Psychology, 1978, 98, 187-195.

Ryan, T.A. Significance tests for multiple comparison of proportions, variances, and other statistics. Psychological Bulletin, 1960, 57, 318-328.

Shapiro, B., & O'Brien, T. Logical thinking in children ages six through thirteen. Child Development, 1970, 41, 823-829.

Smedslund, J. Psychological diagnostics. Psychological Bulletin, 1969, 71, 237-248.

Taplin, J., Staudenmayer, H., & Taddonio, J. Developmental changes in conditional reasoning: linguistic or logical? Journal of Experimental Child Psychology, 1974, 17, 360-373.

Trabasso, T., Isen, A.M., Dolecki, P., McLanahan, A.G., Riley, C.A., & Tucker, T. How do children solve class inclusion problems? In R.S. Siegler (ed.): Children's thinking: what develops? Hillsdale, N.J.: Lawrence Erlbaum, 1978.

Wason, P.C., & Johnson-Laird, P.N. Psychology of reasoning. Cambridge, Mass.: Harvard University Press, 1972.

Winer, G.A. Class inclusion reasoning in children: a review of the empirical literature. Child Development, 1980, 51, 309-328.

Table 1

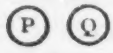
Predicted responses on syllogisms for various interpretations of class relationships within major premise

Syllogism form						
$\frac{P \supset Q \quad P}{\therefore Q}$	(MP)	NO	MAYBE	YES++	YES+	MAYBE
$\frac{P \supset Q \quad \bar{Q}}{\therefore P}$	(MT)	YES	MAYBE	NO++	NO+	MAYBE
$\frac{P \supset Q \quad \bar{P}}{\therefore Q}$	(DA)	YES	MAYBE++	NO	MAYBE+	NO
$\frac{P \supset Q \quad Q}{\therefore P}$	(AC)	NO	MAYBE++	YES	MAYBE+	YES

Note: + = correct response with correct premise interpretation.
++ = correct response, but incorrect premise interpretation.

Table 2

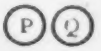
Frequencies of classification and class inclusion responses (expected frequencies in parenthesis)[a].

Classification categories	Class Inclusion			
	"more P"	"more Q"	"P=Q"	total
	100 (50.15)	170 (222.84)	34 (31.00)	304
	36 (43.06)	149 (191.33)	76 (26.62)	261
	13 (6.76)	22 (30.06)	6 (4.18)	41
	45 (134.61)	741 (598.17)	30 (83.22)	816
all other	81 (40.42)	140 (179.60)	24 (24.99)	245
total	275	1222	170	1667

[a] Expected frequencies rounded to nearest one-hundredth.

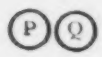
Table 3

Class relations: meanings of major premise and associated frequencies of validity judgments for each of the four argument forms (expected frequencies in parenthesis)[a].

Class relations	"YES"	"NO"	"MAYBE"	total
Modus Ponens				
	13(25.77)	19(5.96)	1(1.27)	33
	49(49.99)	15(11.56)	-(2.46)	64
	11(11.71)	3(2.71)	1(.58)	15
	213(189.01)	16(43.70)	13(9.29)	242
other	60(69.51)	27(16.07)	2(3.42)	89
total	346	80	17	443
Modus Tollens				
Class relations	"YES"	"NO"	"MAYBE"	total
	20(9.74)	16(24.47)	1(2.79)	37
	25(16.58)	37(41.66)	1(4.76)	63
	- (2.63)	7(6.61)	3(.76)	10
	36(60.53)	170(152.11)	24(17.37)	230
other	34(25.52)	59(64.15)	4(7.32)	97
total	115	289	33	437

[a] Expected frequencies rounded to nearest one-hundredth.

Table 3 (cont'd)

Class relations	Denial of the Antecedent "YES"	"NO"	"MAYBE"	total
	17(15.52)	21(14.23)	-(8.24)	38
	32(28.20)	31(25.83)	6(14.97)	69
	6(6.13)	6(5.62)	3(3.25)	15
	84(93.18)	67(85.37)	77(49.46)	228
other	40(35.96)	39(32.95)	9(19.09)	88
total	179	164	95	438

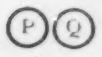
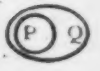
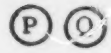
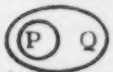
Class relations	Affirmation of the Consequent "YES"	"NO"	"MAYBE"	total
	21(24.19)	21(8.42)	1(10.39)	43
	44(32.62)	9(11.36)	5(14.02)	58
	7(6.75)	5(2.35)	-(2.90)	12
	99(130.50)	30(45.43)	103(56.07)	232
other	99(75.94)	29(26.44)	7(32.62)	135
total	270	94	116	480

Table 4

Frequencies of responses within major categories of understanding for each of the five age groups (proportions in parenthesis)

Category of understanding	grade	frequency	probability
 (exclusion)	K	283(.311)	p<.001
	2	164(.159)	n.s.
	4	171(.159)	p<.01
	6	125(.119)	n.s.
	8	120(.118)	
(equivalence)	K	69(.076)	p<.001
	2	211(.205)	n.s.
	4	210(.195)	p<.01
	6	162(.155)	n.s.
	8	145(.143)	
(intersect)	K	53(.058)	p<.05
	2	40(.039)	p<.01
	4	20(.019)	p<.001
	6	44(.042)	p<.001
	8	14(.014)	
 (inclusion)	K	74(.081)	p<.001
	2	446(.433)	p<.001
	4	623(.579)	p<.01
	6	662(.631)	p<.001
	8	720(.709)	
No response	K	432(.474)	p<.001
	2	168(.163)	p<.001
	4	52(.048)	n.s.
	6	55(.052)	p<.001
	8	16(.016)	